Buffeting of a Slender Circular Beam in Axial Turbulent Flows

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This paper deals with the buffeting of a slender, circular, flexible beam-rod in an axial turbulent flow. The principal excitation mechanisms are the turbulent wall pressure fluctuations and the motion-dependent (self-excited) aerodynamic force caused by the beam motion. On the assumption that the turbulent wall pressure fluctuations are independent of the beam motion, a linear "forced-vibration" model is used to determine the buffeting response of the beam and to investigate the length scale effects of turbulences on the beam buffeting. Transverse buffeting of the beam in an axial turbulent flow depends largely on the ratio of the longitudinal scale of the turbulences to the bending wavelength of the beam and on the ratio of the circumferential scale of the turbulences to the radius of the beam. The spectra and the mean square values of the buffeting displacement of the beam become vanishingly small, both when either of these ratios is very small ($<10^{-2}$) and when the latter is very large ($>10^2$). When the former ratio is very large (i.e., the longitudinal scale of the turbulences is very large compared with the bending wavelength of the beam), the first mode of the buffeting is dominant and other modes are less apparent, especially the high-frequency modes.

Nomenclature

a_{nj}	$= \int_0^L \phi_n''(z)\phi_j(z)\,dz$
A_{nj}	= real part of matrix C_{nj}
b_{nj}	$= \int_0^L \phi_n'(z)\phi_j(z)\,dz$
B_{nj}	= imaginary part of matrix C_{nj}
c	= viscous damping coefficient of the beam
c_I	= speed of sound
C_{nj}	= matrix of system receptance
E	= Young's modulus of elasticity of the beam
F	= self-excited aerodynamic force
H_I	= first kind of Hankel function of order one
I	= area moment of inertia, $i=\sqrt{-1}$
J_0, J_I	= first kind of Bessel functions of order zero and one, respectively
k	= acoustic wave number, ω/c_I
K_c	= convective wave number, ω/U_c
K_n	$= n\pi/L$, bending-vibration wave number
L	= length of the beam
L_I	= longitudinal scale of turbulences
L_2	= circumferential scale of turbulences
m	= mass per unit length of the beam
n	= modal index
p	= turbulent wall pressure fluctuations
p_s	= self-excited pressure
P	= Fourier transform of p
q	= fluctuating aerodynamic force
Q	= Fourier transform of q
r	= radial coordinate
R	= radius of the beam
$S_p(\omega)$	= spectral density of wall pressure at one point
$S_p(z,z',\theta,\theta',\omega)$	= cross-spectral density of p
$S_q(z,z',\omega)$	= cross-spectral density of q
$S_y(z,z',\omega)$	= cross-spectral density of y
t	= time
U	= freestream velocity
U_c	= convective velocity
y	= bending displacement
Y	= Fourier transform of y
Y_0, Y_I	= second kind of Bessel functions of order zero and one, respectively
z	= axial coordinate
γ	$= 1/L_I - iK_c$
δ_{nj}	= delta function
ζ	$= L_I/L$
ζ_n	= modal damping factor, $c/2m\omega_n$
η	$= L_2/R$
θ	= circumferential coordinate
λ	$= kR$
μ	= ratio of fluid density to solid (beam) density, ρ/ρ_s
ξ	$= z' - z$
ρ	= density of fluid
ρ_s	= density of solid (beam)
σ	$= 1/L_I + iK_c$
τ_n	= normal coordinates
ϕ_n	= modal shape functions of free bending-vibration
χ	$= R(\theta' - \theta)$
ω	= circular frequency
ϕ	= velocity potential of disturbed fluid
ω_n	= natural frequency of bending vibration
Ω_n	$= \omega/\omega_n$

Superscripts

$(\ )''$	$= d^2(\ )/dz^2$
$(\ )'$	$= d(\ )/dz$
$(\ )^*$	= complex conjugate

Subscripts

c	= convective
j,n	= modal indices
p	= pressure
q	= fluctuating force
y	= displacement

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Introduction

HISTORICALLY, study of airfoil buffeting goes back to a 1930s investigation of an English airplane accident.[1] Since then many studies have been made of the buffeting and gust response of aircraft in turbulent winds; see, e.g., Refs. 2-10 and their reference lists. Liepmann[4] was the first to treat statistically the problem of airfoil buffeting. He considered a two-dimensional airfoil of a single degree of freedom in an isotropic, homogeneous turbulence field and in a vortex-street flow. In 1955, Liepmann[6] extended his two-dimensional theory of buffeting to a three-dimensional one, in which the three-dimensional fluctuations were taken into account. In his paper on statistical aspects of dynamic loads, Fung[5] also treated statistically a rigid airplane to the atmospherical turbulences and obtained the root mean square value of the airplane acceleration caused by gusty turbulences.

In the past three decades, the understanding of aeroelasticity has developed rapidly in many applications, both aeronautical and nonaeronautical. Aeroelasticity in aeronautical fields includes the determination of unsteady aerodynamic loadings on and the structural response of airfoils, fuselages, missiles, rockets, and spacecraft components. Nonaeronautical aeroelasticity encompasses many areas in civil, chemical, mechanical, nuclear, and naval architecture engineering. Among all of the fields of aeroelastic interests, cylindrical structures play very important and vital roles. For instance, the slender bodies of aircraft fuselages, missiles, and rockets and the main bodies of industrial smokestacks, power transmission cables, oil pipelines, reactor fuel rods, heat exchanger tubes, and off-shore structures are primarily made up of slender circular cylindrical shells. These structural components are subject to unsteady aerodynamic (or hydrodynamic) forces capable of causing deformation when they are in operation. Therefore, aeroelasticians are all concerned with the deformation characteristics of slender, circular cylindrical shells vibrating in unsteady flows.

The study of unsteady fluid dynamic forces on cylindrical bodies moving in axial flows (i.e., the mean flows are parallel to the longitudinal axes of the bodies) became of interest in the 1950s when Sir Geoffrey Taylor[11] developed a flexible, long cylinder that he set obliquely to a stream of fluid to simulate the swimming of long, narrow animals. References 12-16 outline the hydrodynamics of the swimming propulsion by a flexible, infinite cylinder (representing a slender aquatic animal) and a two-dimensional flexible plate (representing the flapping wing of a bird or an insect). Sir James Lighthill thoroughly investigated the hydrodynamic forces on a slender fish moving in an incompressible flow,[12] based on the slender-body theory.[17] The detailed and extensive investigations of cylindrical structures vibrating in an axial flow started in the late 1950s. Recent surveys[18,19] of subjects involving the flow-induced vibrations of cylinders reference more than 200 papers. One-quarter of these 200 deal with the external, axial-flow-induced vibrations of a single cylinder or bundle of cylinders. Most of them consider the aeroelastic (or hydroelastic) oscillations of cylinders (including shells) in laminar flows. To the best knowledge of the present author, there has been no formative and rigorous analysis of the buffeting of an elastic cylinder with multiple degrees of freedom in an axial turbulent flow. The present study was undertaken to study analytically the buffeting of a slender, flexible, circular cylinder in axial turbulent flows and to examine the scalar effects of turbulent eddies on the buffeting of the cylinder.

Considered here is a long, flexible, circular cylinder immersed in an axial turbulent flow, as shown in Fig. 1. For the sake of mathematical simplicity and the physical condition of the slenderness of the cylinder, the simple "slender-beam" theory is used to describe the small motion of the cylinder and an axially convective flow model to describe the turbulent motion in the present study. The main excitations are the turbulent wall pressure fluctuations and the motion-dependent aerodynamic forces. The turbulence field in the flow is assumed not to severely modify the motion of the cylinder; thus, a linear forced-vibration model is used to determine the buffeting displacement of the cylinder. The fluctuating wall pressures are considered as purely external random forcing mechanisms with homogeneously spatial distribution, and the motion-dependent aerodynamic forces are approximated by the slender-body theory.[17,20-24] Based on the theory of random vibration, the buffeting response of the cylinder is formulated in terms of the spectral density of the wall pressure fluctuations and the receptance of the aeroelastic system.

Equations of Motion

For simplicity, the following assumptions are made to formulate the equation describing the buffeting of the cylinder in an axial turbulent flowfield: 1) the fluid is assumed to be inviscid, so that the viscous force can be neglected; 2) the cylinder motion due to fluctuating wall pressures beneath the turbulent boundary layer is sufficiently small so the linear beam theory is applicable; 3) the interaction between the turbulent flowfield and the cylinder motion is negligible. Buffeting of the cylinder in the turbulent flow is then considered as the dynamic response of a linear system to the random wall pressures. The equation of small transverse motion of the cylinder is

$$EI\frac{\partial^4 y}{\partial z^4}+c\frac{\partial y}{\partial t}+m\frac{\partial^2 y}{\partial t^2}=q(R,\theta,z,t)+F(R,y,z,t) \tag{1}$$

The motion-dependent aerodynamic force $F(R,y,z,t)$, as obtained by the slender-body theory outlined in the Appendix, is

$$F(R,y,z,t)=-\pi R^2\rho(\alpha+i\beta)\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial z}\right)^2 y \tag{2}$$

where

$$\alpha=\frac{\lambda[J_0(\lambda)J_1(\lambda)+Y_0(\lambda)Y_1(\lambda)]-J_1^2(\lambda)-Y_1^2(\lambda)}{[\lambda J_0(\lambda)-J_1(\lambda)]^2+[\lambda Y_0(\lambda)-Y_1(\lambda)]^2} \tag{3}$$

$$\beta=\frac{\lambda[J_0(\lambda)Y_1(\lambda)-Y_0(\lambda)J_1(\lambda)]}{[\lambda J_0(\lambda)-J_1(\lambda)]^2+[\lambda Y_0(\lambda)-Y_1(\lambda)]^2} \tag{4}$$

The fluctuating aerodynamic force $q(R,\theta,z,t)$ is related to the turbulent wall pressures $p(R,\theta,z,t)$ on the cylinder surface as

$$q(R,z,t)=-\int_0^{2\pi} p(R,\theta,z,t)R\cos\theta d\theta \tag{5}$$

Substituting the explicit forms of the motion-dependent and fluctuating aerodynamic forces into Eq. (1), we have an equation of motion for the aeroelastic system of the slender cylinder and the axial turbulent flow,

$$\begin{aligned}EI\frac{\partial^4 y}{\partial z^4}&+\left[c+2U\pi R^2\rho(\alpha+i\beta)\frac{\partial}{\partial z}\right]\frac{\partial y}{\partial t}\\&+[m+\pi R^2\rho(\alpha+i\beta)]\frac{\partial^2 y}{\partial t^2}+\pi R^2\rho(\alpha+i\beta)U^2\frac{\partial^2 y}{\partial z^2}\\&=-\int_0^{2\pi}p(R,\theta,z,t)R\cos\theta d\theta\end{aligned} \tag{6}$$

Forced Vibration of the Cylinder

The basic features of the aeroelastic system, of which the slender cylinder is the key element, are the interaction of the cylinder motion and fluid motion consisting of the mean flow, freestream turbulence, boundary-layer turbulence, and wake turbulence (if any). On the assumption that the cylinder motion does not alter the characteristics of the turbulences in the flowfield, the buffeting response of the beam can be regarded as the forced vibration due to the fluctuating wall

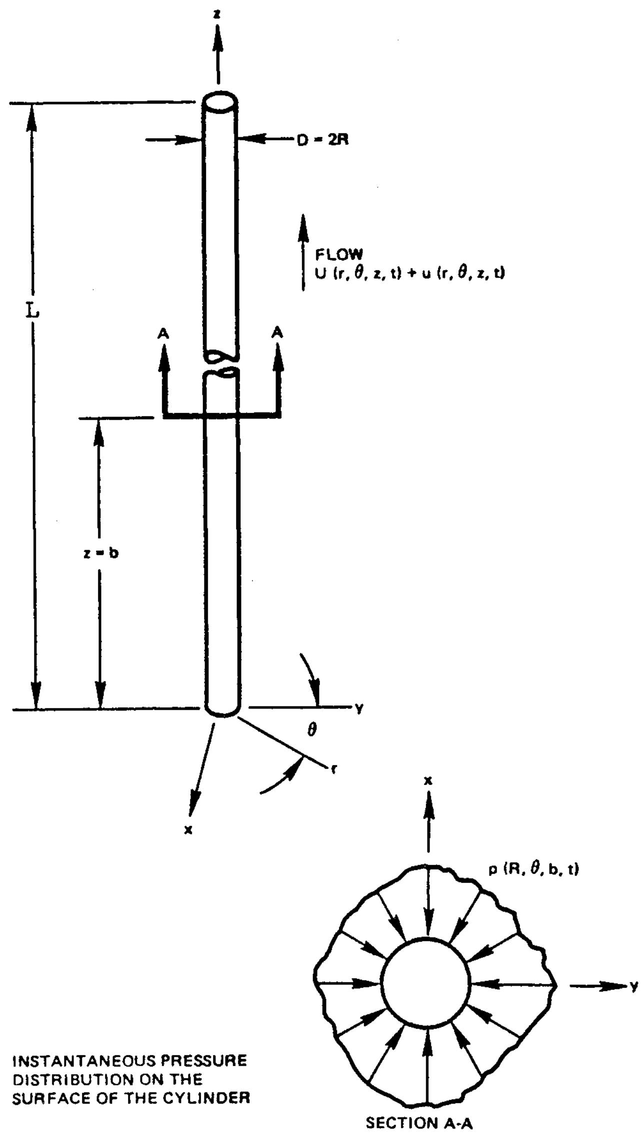

Fig. 1 Slender, flexible, circular beam in a turbulent flow and its coordinate system.

pressures. The mathematical model shown in Eq. (6) for the cylinder buffeting is therefore nonautonomous.

Assume $p(R,\theta,z,t)$ is a stationary random process with zero mean and is continuous in the quadratic mean sense; then its Fourier transform is a random process defined by the quadratic mean integral

$$\int_{-\infty}^{\infty} p(R,\theta,z,t)\,e^{i\omega t}\mathrm{d}t = P(R,\theta,z,\omega) \tag{7}$$

Similarly, we can define the Fourier transforms of $q(R,z,t)$ and $y(z,t)$ as

$$\int_{-\infty}^{\infty} q(R,z,t)\,e^{i\omega t}\mathrm{d}t = Q(R,z,\omega) \tag{8}$$

and

$$\int_{-\infty}^{\infty} y(z,t)\,e^{i\omega t}\mathrm{d}t = Y(z,\omega) \tag{9}$$

According to modal analysis, we assume the solution of Eq. (6) has the form

$$y(z,t) = \sum_n \phi_n(z)\,\tau_n(t) \tag{10}$$

Substituting this modal representation of $y(z,t)$ into Eq. (6), taking the Fourier transformation of the normal coordinates, and using the orthogonality condition of the normal modes, we obtain

$$Y(z,\omega) = \sum_n \sum_j \phi_n(z)\,[C_{nj}]^{-1} \int_0^L Q(z,\omega)\,\phi_j(z)\,\mathrm{d}z \tag{11}$$

where C_{nj} is a complex matrix representing the system receptance. Its real and imaginary parts are, respectively,

$$A_{nj} = \omega_n^2\left[\,(1-\Omega_n^2-\alpha\mu\Omega_n^2)\,\delta_{nj} + \alpha\mu a_{nj}U^2/\omega_n^2 - 2\beta\mu U b_{nj}\Omega_n/\omega_n\,\right] \tag{12}$$

$$B_{nj} = \omega_n^2\left[\,(2\zeta_n\Omega_n-\beta\mu\Omega_n^2)\,\delta_{nj} + \mu a_{nj}\beta U^2/\omega_n^2 + 2\alpha\mu U b_{nj}\Omega_n/\omega_n\,\right] \tag{13}$$

The spectral density of buffeting displacement $y(z,t)$ is then obtained as

$$S_y(z,z',\omega) = \sum_n \sum_j \sum_m \sum_k \phi_n(z)\,\phi_m(z')\,[C_{nj}C^*_{mk}]^{-1} \times \int_0^L \int_0^L S_q(z,z',\omega)\,\phi_j(z)\,\phi_k(z')\,\mathrm{d}z\mathrm{d}z' \tag{14}$$

Assume that the damping of the aeroelastic system is small enough so that each mode of the cylinder vibration is distinct (i.e., no modal overlapping). The cross-product terms of $S_y(z,z',\omega)$ are then negligible and Eq. (14) simplifies to

$$S_y(z,z',\omega) = \sum_n \sum_j \phi_n(z)\,\phi_n(z')\,[C_{nj}C^*_{nj}]^{-1} \times \int_0^L \int_0^L S_q(z,z',\omega)\,\phi_j(z)\,\phi_j(z')\,\mathrm{d}z\mathrm{d}z' \tag{15}$$

In terms of cross-spectral density of the wall pressure fluctuations, the spectral density of the buffeting displacement of the cylinder becomes

$$S_y(z,z',\omega) = \sum_n \sum_j \phi_n(z)\,\phi_n(z')\,[C_{nj}C^*_{nj}]^{-1}R^2 \times \int_0^L \int_0^L \phi_j(z)\,\phi_j(z') \int_0^{2\pi} \int_0^{2\pi} S_p(z,z',\theta,\theta',\omega) \times \cos\theta\cos\theta'\,\mathrm{d}\theta\mathrm{d}\theta'\,\mathrm{d}z\mathrm{d}z' \tag{16}$$

Effects of Turbulence Scales on Buffeting

To show the effects of turbulence length scales on the buffeting of the cylinder, we assume that the fluctuating wall pressures are spatially homogeneous. For a spatially homogeneous and convective wall pressure field, the cross-spectral density is[25]

$$S_p(z,z',\theta,\theta',\omega) = S_p(\omega)\exp\left[-|\xi|/L_1 - iK_c\xi - |\chi|/L_2\right] \tag{17}$$

Substituting Eq. (17) into Eq. (16) and integrating with respect to the angular coordinate, we have the spectral density of the buffeting displacement of the cylinder,

$$S_y(z,z',\omega) = \sum_n \sum_j \phi_n(z)\,\phi_n(z')\,[C_{nj}C^*_{nj}]^{-1}S_p(\omega)\,G(\eta)\,R^2 \times \int_0^L \int_0^L \phi_j(z)\,\phi_j(z')\exp(-|\xi|/L_1 - iK_c\xi)\,\mathrm{d}z\mathrm{d}z' \tag{18}$$

where

$$G(\eta)=\frac{2\eta}{1+\eta^2}\left[\pi+\frac{\eta}{1+\eta^2}\left(e^{-2\pi/\eta}-1\right)\right] \tag{19}$$

Up to this point, we have obtained in a double integral form the general expression for the spectral density of the cylinder buffeting. In the following, we focus on the case of a simply supported cylinder beam to simplify the mathematical treatment. For a simply supported beam, the normal modes of free bending vibrations are

$$\phi_n(z)=\sqrt{2/mL}\sin(K_n z),\qquad n=1,2,3,\ldots \tag{20}$$

With these modal functions, it can be easily proved that the coefficients a_{nj} and b_{nj} are, respectively, $-(n\pi)^2\delta_{nj}/mL^2$ and zero. Hence, the matrix elements of the system receptance become diagonal, namely,

$$A_{nj}=\omega_n^2[1-\Omega_n^2-\alpha\mu\Omega_n^2-\alpha\mu U^2K_n^2/m\omega_n^2]\delta_{nj} \tag{21}$$

$$B_{nj}=\omega_n^2[2\zeta_n\Omega_n-\beta\mu\Omega_n^2-\beta\mu U^2K_n^2/m\omega_n^2]\delta_{nj} \tag{22}$$

The double integral of Eq. (18) with the modal function of Eq. (20) can be integrated, with the result

$$I(m,K_n,K_c,L_1)=\frac{1}{m}\left[\frac{\gamma}{\gamma^2+K_n^2}+\frac{\sigma}{\sigma^2+K_n^2}\right]$$
$$+\frac{2K_n^2}{mL}\left[\frac{1+(-1)^{n+1}e^{-\gamma L}}{[\gamma^2+K_n^2]^2}+\frac{1+(-1)^{n+1}e^{-\sigma L}}{[\sigma^2+K_n^2]^2}\right] \tag{23}$$

Therefore, the spectral density of the buffeting displacement of a simply supported beam in bending modes becomes

$$S_y(z,z',\omega)=\frac{2R^2}{mL}G(\eta)S_p(\omega)\sum_n\sum_j[C_{nj}C_{nj}^*]^{-1}$$
$$\times I(m,K_n,K_c,L_1)\sin(K_nz)\sin(K_jz') \tag{24}$$

The influence of the axial correlation length of turbulences on the spectral density of the beam buffeting can be seen from the variation of the dimensionless factor $mI(m,K_nK_c,L_1)/2L$ with respect to K_nL_1. Figure 2 shows the variation of $mI/2L$ with respect to the ratio of L_1/L (ζ) for $K_c=0$, namely, no convection. The numerical values of $mI/2L$ become very small (≈ 0), both when $\zeta<10^{-2}$ and when $\zeta>10$ for all modes, except for the first mode ($n=1$) when $\zeta>10$. The value of $mI/2L$ is a constant (0.4052) for $\zeta>10$. For the general case of $K_c\neq 0$, the factor $I(m,K_n,K_c,L_1)$ becomes

$$I(m,K_n,K_c,L_1)=\frac{2L_1/m}{1+(K_cL_1)^2} \tag{25}$$

when the wavelength of the bending vibration of the beam is much greater than the longitudinal scale of the turbulence ($K_nL_1\to 0$). The spectral density of beam buffeting then reduces to

$$S_y(z,z',\omega)=\frac{4\zeta/m^2}{1+(K_cL_1)^2}G(\eta)S_p(\omega)R^2$$
$$\times\sum_n\sum_j[C_{nj}C_{nj}^*]^{-1}\sin(K_nz)\sin(K_jz') \tag{26}$$

This expression shows that the cross-spectral density of the beam buffeting is proportional to the ratio of L_1/L, which is very small, as Fig. 2 shows, for ζ being very small. Therefore, the spectra and mean square value of the buffeting beam are very small as well. This result shows that the large-wavelength motion of the beam (i.e., low-frequency vibration) does not experience the net effect of small turbulent eddies.

On the other hand, when the wavelength of the bending vibration of the beam is much smaller than the longitudinal scale of the turbulence, i.e., $K_nL_1\to\infty$, the factor $I(m,K_n,K_c,L_1)$ becomes

$$I(m,K_n,K_c,L_1)=\frac{4L}{m(n\pi)^2}[1+(-1)^{n+1}\cos(K_cL)] \tag{27}$$

and the spectral density of the buffeting beam becomes

$$S_y(z,z',\omega)=G(\eta)S_p(\omega)\sum_n\sum_j[C_{nj}C_{nj}^*]^{-1}\frac{8R^2}{m^2n^2\pi^2}$$
$$\times[1+(-1)^{n+1}\cos(K_cL)]\sin(K_nz)\sin(K_jz') \tag{28}$$

When the modal index n becomes large (corresponding to the high-frequency mode), the value of $I(m,K_n,K_c,L_1)$ becomes small. Hence, the high-frequency modes make little contribution to the buffeting of the beam. That is, the high-frequency vibrations of the beam do not perceive the randomness of the large-scale turbulences. This phenomenon is also seen in Fig. 2.

For the special case of no convection ($K_c=0$), the factor I becomes

$$I(m,K_n,K_c,L_1)=\frac{4L}{m(n\pi)^2}[1+(-1)^{n+1}] \tag{29}$$

When the modal index n is even, the values of I are zero, as is the spectral density of the buffeting. On the other hand, when

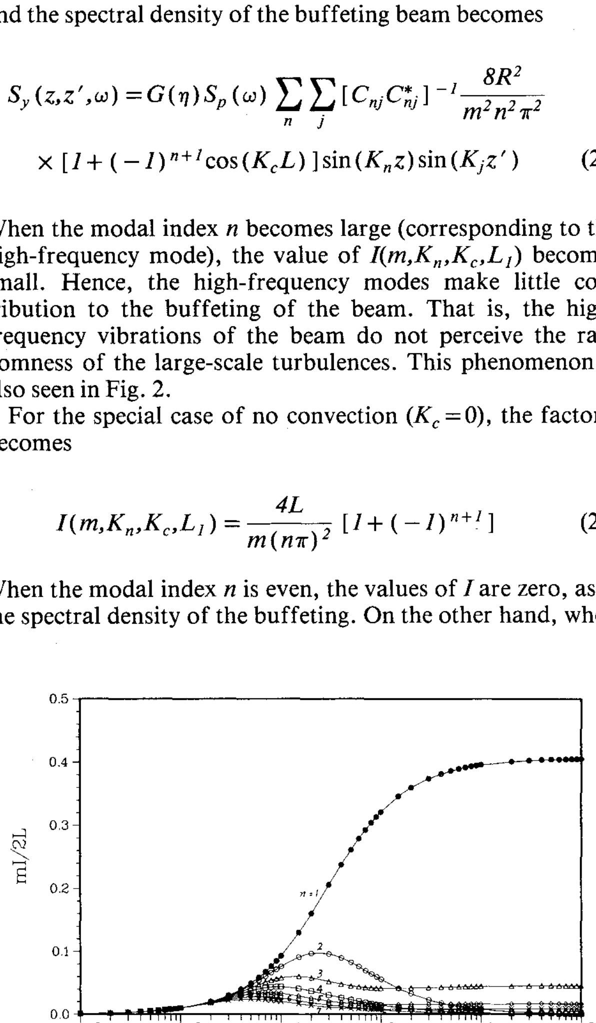

Fig. 2 Dimensionless factor $mI/2L$ vs the ratio of L_1/L.

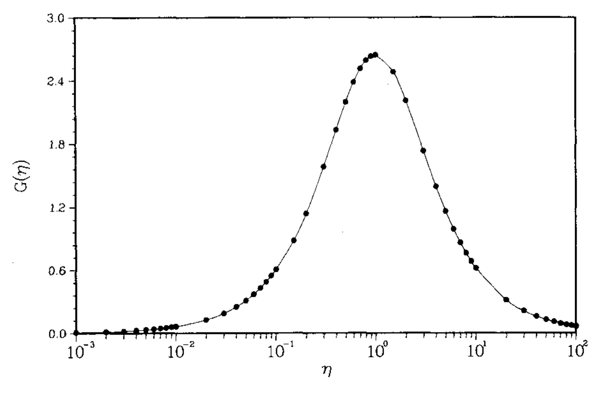

Fig. 3 Circumferential influence function $G(\eta)$ vs the ratio of L_2/R.

n is odd, the value of I is $8L/m(n\pi)^2$, so that the higher odd modes of the beam vibrations make very minute contributions to the buffeting.

Similarly, we can investigate the effect of the circumferential length of the turbulence on beam buffeting by comparing the values of L_2 and $2\pi R$. Figure 3 shows the variation of $G(\eta)$ with respect to η (the ratio of L_2/R). For both $\eta<10^{-2}$ and $\eta>10^2$, the numerical values of the dimensionless factor $G(\eta)$ are very small and approach zero. Hence, the spectra and the mean square value of the buffeting beam are zero when $\eta\rightarrow 0$ and $\eta\rightarrow\infty$. This phenomenon is very similar to the effect of the axial correlation length and the same reasoning is applied here. However, the circumferential correlation has a maximum when the circumferential scale of turbulences is the same as the radius of the beam.

Conclusion

This paper presents an analytical investigation of the buffeting of a slender beam of circular cross section in an axial turbulent flow. The analysis provides a simple, but rigorous, formulation to determine the buffeting response of a slender beam in two-dimensional turbulent flows and to investigate the effects of turbulence scales on beam buffeting.

The results show that the transverse buffeting of a slender flexible beam in axial turbulent flows greatly depends on the ratio of the longitudinal scale of the turbulences to the wavelength of the beams's bending vibration and on the ratio of the circumferential scale of the turbulences to the circumference of the beam. When either of these ratios is vanishingly small, corresponding to the case of vibration (especially low-frequency vibration) of the beam in small-scale turbulence fields, the buffeting response of the beam is proportional to that ratio and hence is negligible. On the other hand, when either of the ratios is infinitely large (corresponding to the cases of beam vibrations in a large-scale turbulence field), the circumferential effect of turbulence is very small and the longitudinal effect of turbulence is dominant in the first mode. Large-scale turbulences have a very minute effect on the other modes. Therefore, the buffeting response caused by high-frequency vibrations in large-scale turbulences is vanishingly small. All the above results are quite similar to those of the gust response of a rigid airplane to atmospheric turbulences, as reported in Ref. 5. This is not surprising, because the rigid airplane is an ideal model of the elastic airplane and the atmospheric gust reveals random, turbulent motion.

When the wavelength of the bending vibration of the beam is very large compared with the longitudinal scale of turbulence, the flowfield seems to be smooth with respect to that particular mode of vibration. Thus, the beam responds to the null effects of the turbulences. On the other hand, if the bending vibration wavelength of the beam is quite small compared with the longitudinal scale of the turbulence, a turbulent eddy covers many wavelengths in the bending vibration of the beam, so that the flowfield seems to be stationary with respect to these modes of vibration and the beam behaves in a quasistationary manner, as it does in laminar flow. The unsteady turbulences do not have much influence on the beam vibrations, except in the first mode.

In many practical cases of aeronautical structures in flight, both limiting cases are possible, because the size of the separated vortices or turbulences associated with the flight covers a wide range of length scales. The most important situation is that when the bending vibration wavelengths of the structure components are comparable with the longitudinal lengths of the turbulent eddies or when the radii of the structure components are close to the circumferential lengths of the turbulences, the buffeting responses from all modes (especially the first mode) of the structure components are dominant and severe. The total buffeting response of the structure is the summation of the individual contribution to buffeting from each mode of the structure-component bendings.

Appendix: Self-Excited Aerodynamic Force

Early in the 1960s, Sir James Lighthill applied the slender-body theory to treat the problem of a slender fish swimming in an inviscid, incompressible flow[12] and obtained the instantaneous lift per unit length of the fish and the work rate done by the fish movements. He also considered the boundary-layer effects on the flow and the dynamics of the fish movements. In 1971, Wu also used the slender-body approximation to investigate the hydrodynamics of swimming propulsion of a slender fish with side fins in an inviscid incompressible flow,[15] in which the shedding of vortex sheets from sharp trailing edges was considered in great detail.

Recently, Paidoussis and Ostoja-Starzewski employed the slender-body theory to determine the self-excited aerodynamic force on a circular, flexible cylinder in a circular duct of compressible flow.[26] They have obtained two- and three-dimensional solutions for the aerodynamic forces; however, they neglected the possible effects of vortex sheddings resulting from steep gradients of the displaced boundary on the flow and on the dynamics of the cylinder. This kind of approach is permissible only if the cylinder is very long, such that the disturbed flow caused by the motion of the cylinder is two-dimensional, i.e., with no axial dependence. The same approach is used in the following to determine the self-excited aerodynamic force on a circular beam-rod vibrating in an unbounded compressible flow.

The velocity potential of the disturbed-fluid motion caused by the small motion of the cylinder beam, as Fig. 1 shows, is governed by

$$\nabla^2\phi=\frac{l}{c_l^2}\frac{\partial^2\phi}{\partial t^2} \tag{A1}$$

and the boundary conditions by

$$\frac{\partial\phi}{\partial r}\bigg|_{r=R}=\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial z}\right]y\cos\theta \tag{A2}$$

and ϕ is outgoing.

The complete solution of Eq. (A1) in two-dimensional (r,θ) coordinates is

$$\phi=\frac{H_l(kr)}{H_l'(\lambda)}\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial z}\right]y\cos\theta \tag{A3}$$

for harmonic disturbances. The pressure caused by the disturbed flow is given by the linearized formula

$$p_s=-\rho\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial z}\right]\phi \tag{A4}$$

The aerodynamic force on the beam per unit length is the integration of the pressure around the beam circumference. That is,

$$F=\int_0^{2\pi}p_sR\cos\theta d\theta \tag{A5}$$

Substituting ϕ into this equation and perfoming the integration, we have

$$F=-\pi R^2\rho(\alpha+i\beta)\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial z}\right]^2y \tag{A6}$$

with α and β being defined in Eqs. (3) and (4). This simple solution does not take into consideration the effects of the variation in the beam cross section and the modal shapes of the beam on the unsteady aerodynamics. To account for the modal shape effect on the unsteady aerodynamic force, the methods shown in Refs. 26 and 27 are appropriate.

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